Kyla Orchard

MATHS NOTES - COUNTING + PROBABILITY

SETS + VENN DIAGRAMS

absolute value → $|C|$, $n(C)$ - how many elements/members in set C?

$|D| = 5$ - set D has 5 elements/members

$X = \{2, 4, 5\}$ - X contains 2, 4, 5

$3 \in B$ - 3 is an element/member of set B

$3 \notin B$ - 3 isn't an element/member of set B

$A \subset E$ - set A is contained (is a subset) of set E

$A \not\subset E$ - set A isn't contained in set E

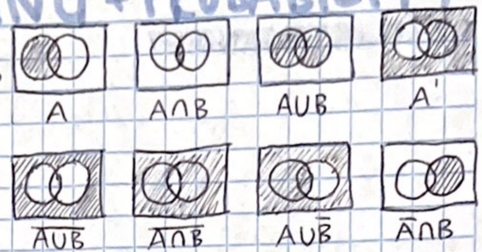
$P \cup Q$ - P union Q

$P \cap Q$ - P intersection/overlap Q

A', $\bar{A}$ - compliment of set A, not in set A

U - universal set

$\emptyset$ - zero set, a set containing 0 elements



ARRANGEMENTS vs COMBINATIONS

Arrangements

also called permutations

e.g. 'ABC' can be written 6 ways when arranged:

ABC, ACB, BAC, BCA, CAB, CBA

can be found using the multiplication principle:

$\boxed{3 \times 2 \times 1}$

i.e. $3!$ ← factorial

$\frac{100!}{99!} = 100$ $\quad \frac{100 \times \cancel{99} \times \cancel{98}}{\cancel{99} \times \cancel{98} \times \cancel{97}}$ (they cancel)

Combinations

e.g. ABCD, when choosing 2 nos there are 6 ways (because AB and BA are the same)

formula for ^nC_r

$$^nC_r = \frac{n!}{r!(n-r)!}$$

n = how many you have (no. of items)

r = how many you want

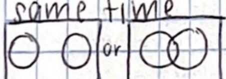
e.g. $^6C_4 = \frac{6!}{4!2!}$

or with calc: nCr(6, 4)

$^nC_r = \binom{n}{r}$

MUTUALLY EXCLUSIVE

- cannot occur at the same time



but $A \cap B = 0$ (zero)

'NOT' 'AND' + 'OR'

not: same as compliment.

not a = 1 - a

and: same as intersection

a and b = $A \cap B$

or: same as union

a or b = $A \cup B$

PROBABILITY

$$P(\text{event}) = \frac{\text{no. of successful outcomes}}{\text{total no. of outcomes}}$$

venn diagram

2-way table

tree diagram

CONDITIONAL PROBABILITY

$P(A|B)$ = $\frac{A \text{ in } B}{\text{all of } B}$

↑ given that

PASCAL'S TRIANGLE (full on back)

							1							
						1		1						
					1		2		1					
				1		3		3		1				
			1		4		6		4		1			
		1		5		10		10		5		1		
	1		6		15		20		15		6		1	
1		7		21		35		35		21		7		1

expand $(x+y)^5$

= check row 1 after

$= 1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5$

x's go 5, 4, 3, 2, 1, 0

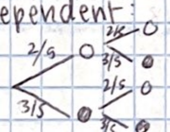
y's go 0, 1, 2, 3, 4, 5

$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

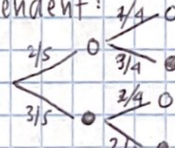
INDEPENDENCE

indepent = the occurrence of one doesn't affect the probability of the other

independent:



dependent:



PASCAL'S TRIANGLE

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1

1 11 55 165 330 462 462 330 165 55 11 1

1 12 66 220 495 792 924 792 495 220 66 12 1

trends:

- on rows beginning with primes, all numbers in that row are divisible equally by that first number
- if you add all the numbers in a row together, it becomes a power of 2 (row 1=1= 2^0, row 2 = 2 = 2^1, row 3 = 4 = 2^2 etc.)
- putting all the numbers together becomes a power of 11 (row 2 = 11 = 11^1, row ~~4~~ = 1331 = 11^2, row 4 = 14641 = 11^3)

PINK SHEET

- complementary events (A and A')

$$\boxed{P(A') = 1 - P(A)}$$

- conditional probability (B|A)

$$\boxed{P(B|A) = \frac{P(A \cap B)}{P(A)}}$$

- A and B ($A \cap B$)

to determine the probability of A and B occurring we multiply the probabilities together, paying due regard to whether the occurrence of one of the events affects the likelihood of the other occurring:

$$\boxed{P(A \cap B) = P(A) \times P(B|A)}$$

if A and B are independent events, $P(B|A) = P(B)$ and so

$$\boxed{P(A \cap B) = P(A) \times P(B)}$$

- A or B ($A \cup B$)

to determine the probability of A or B occurring we add the probabilities together and then make the necessary subtraction to compensate for the "double counting of the overlap"

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

if A and B are mutually exclusive events, $P(A \cap B) = 0$ and so

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

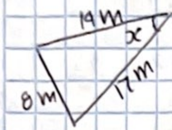
kyla orchard

MATHS NOTES - ~~trigonometry~~

CHECK CALC - rads or deg

SOHCAHTOA

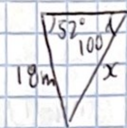
tan is also $\frac{\sin}{\cos}$



$c^2 = a^2 + b^2 - 2ab\cos C$ — $\boxed{\text{cosine rule}}$

$8^2 = 17^2 + 14^2 - 2(17)(14)\cos(x)$

$x = 27.81562176 \approx 28°$

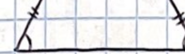


$\frac{x}{\sin(52)} = \frac{18}{\sin(100)}$ — $\boxed{\text{sine rule}}$

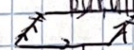
$x = 14.40300761$

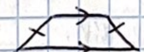
$x = 14m$

isosceles trapezium



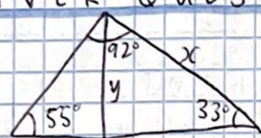
parallelogram





trapezium

RIVER QUESTION



$\frac{x}{\sin(55)} = \frac{120}{\sin(92)}$

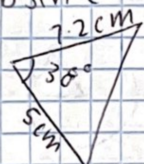
$x = 98.358$

$\sin(33°) = \frac{y}{98.358}$

$y = 53.569 \therefore 54m$

AREA

$\frac{1}{2}ab\sin C$

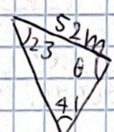


$A = \frac{1}{2}ab\sin C$ — degrees

$= \frac{1}{2}(7.2)(5)\sin(38)$

$= 11.0819$

$= 11.1cm^2$



$180 - 23 - 44 = \theta$

$\theta = 116°$

$A = \frac{1}{2}(52)(41)\sin(116)$

$= 869.6\ m^2$

RADIANS

$\frac{4\pi}{3} \rightarrow x°$

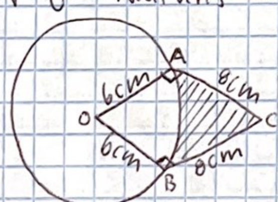
$\frac{\pi}{3} = 60$

60×4

$= 240°$

COMPLEX AREA

$A = r^2\theta$ — radians



A of quad - A of sect

$\tan\theta = \frac{8}{6}$

$\theta = 0.9272$

$\angle AOB = 2 \times 0.9272$

$= 1.8545$

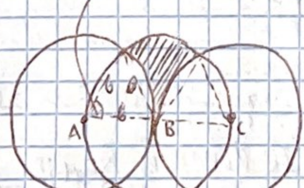
$= 2\left[\frac{1}{2} \times 6 \times 8\right] - \frac{1}{2} \times 6^2 \times 1.8545$

$= 48 - 33.3826$

$= 14.617$

$= 14.6cm^2$

$\theta = 60° = \frac{\pi}{3}$



~~perimeter~~ area:

$A = $ sector(centre B) - segment(centre A) - segment (centre C)

$= \frac{1}{2}r^2\theta - \cancel{2}\left(\frac{1}{\cancel{2}}r^2(\theta - \sin\theta)\right)$

$= \frac{1}{2}6^2\left(\frac{\pi}{3}\right) - 6^2\left(\frac{\pi}{3} - \sin\left(\frac{\pi}{3}\right)\right)$

$= \frac{6^{\cancel{2}}\pi}{\cancel{6}} - 6^{\cancel{2}}\left(\frac{2\pi - 3\sqrt{3}}{\cancel{6}}\right) = 6\pi - 12\pi + 18\sqrt{3}$

$= 18\sqrt{3} - 6\pi\ cm^2 \quad (= 12.33cm^2)$

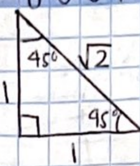
$\frac{2\pi}{\cancel{8}6} - \frac{3\sqrt{3}}{\cancel{2}6} = \frac{2\pi - 3\sqrt{3}}{6}$

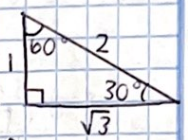
perimeter: $P = 3 \times$ arc length

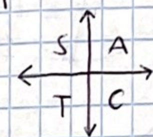
$= 3(r\theta) = 3 \times 6 \times \frac{\pi}{3}$

$= 6\pi = 18.85cm$

BOOKMARK







$\cos = x$

$\sin = y$

$\sin(0°) = 0$

$\sin(90°) = 1$

$\sin\left(\frac{\pi}{2}\right) = 1$

$\sin(180°) = 0$

$\sin(\pi) = 0$

$\cos(0°) = 1$

$\cos(90°) = 0$

$\cos\left(\frac{\pi}{2}\right) = 0$

$\cos(180°) = -1$

$\cos(\pi) = -1$

$\theta = rad \times \frac{180}{\pi}$

$rad = \theta \times \frac{\pi}{180}$

$180° = \pi$

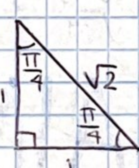
$360° = 2\pi$

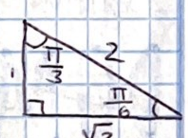
$90° = \frac{\pi}{2}$

$30° = \frac{\pi}{6}$

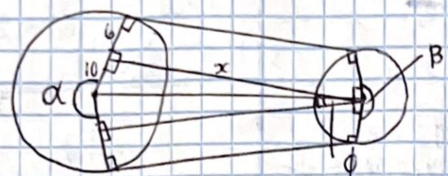
$60° = \frac{\pi}{3}$

$45° = \frac{\pi}{4}$





WHEEL QUESTION

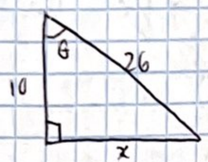


$\tan(\phi) = \frac{10}{24}$ $\quad \phi = 0.39479$

$\beta = 2\pi - 2\phi = 2\pi - 2(0.3479)$
$= 2.352$

ℓ (major of small) $= 6 \times 2.352$
$= 14.112$

total $= 2x + \ell(\text{big}) + \ell(\text{small})$
$= 2 \times 24 + 62.89872 + 14.112$
$= 125\text{cm}$



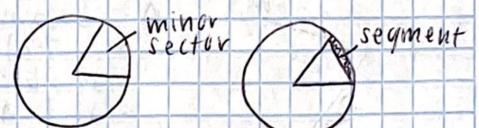
$\tan\theta = \frac{x}{26}$
$= \frac{24}{26}$

$\theta = 1.176$ radians
$\alpha = 2\pi - 2\theta$
$= 2\pi - 2(1.176)$
$= 3.93117$ radians

ℓ (major of big wheel)
$= r\theta = 16 \times 3.93117$
$= 62.89872$

θ = radians

arc length = $(\ell = r\theta)$
sector area = $(\frac{1}{2} r^2 \theta)$
length of chord = $(\ell\, 2r\sin\frac{1}{2}\theta)$
area of seg. = $A = \frac{1}{2} r^2 (\theta - \sin\theta)$



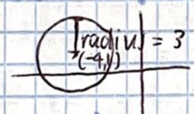
circumference = $2\pi r$
area = πr^2
area of sec or seg = $\frac{\theta}{360} \times 2\pi r$

CIRCULAR RELATIONS

circular relations

$x^2 + y^2 = r^2$
$(x+a)^2 + (x+b)^2 = r^2$ — radius

flip the sign = midpoint — radius
$(x+4)^2 + (y-1)^2 = 3^2$



e.g centre @ (3,5) and r = 5
$(x-3)^2 + (y-5)^2 = 5^2$
$x - 6x + 9 + y^2 - 10y + 25 = 25$
$x^2 + y^2 - 6x - 10y = 25 - 25 - 9$
$x^2 + y^2 6x - 10y = -9$

e.g 2 $\quad x^2 + y^2 + 6y = 10x$
$x^2 - 10x + 25 + y^2 + 6y + 9 = 0 + 25 + 9$ → complete the square
$(x-5)^2 + (y+3)^2 = 34$
$= (\sqrt{34})^2$

∴ centre @ (5, −3)
radius = $\sqrt{34}$

DOMAIN + RANGE

domain + range

Domain = x-values that the function of x & f(x) exists in $\quad \{x \in \mathbb{R}, \_\_\}$
points to consider:
does f(x) involve:
↳ finding a square root of x? — can't determine $\sqrt{x}$
↳ division of x-value? — can't divide by zero

$\{x \in \mathbb{R}\}$ $\quad$ $x = 4$ $\{x \in \mathbb{R}, x \neq 4\}$ $\quad$ $\{x \in \mathbb{R}\}$ $\quad$ $\{x \in \mathbb{R}, x \geq 0\}$

Range = y-values that the function work in based on domain $\quad \{y \in \mathbb{R}, \_\_\}$
points to consider:
↳ the domain, this can help to work it out $\quad \{y \in \mathbb{R}, y \neq 0\}$
↳ $\sqrt{x}$ or $\frac{1}{x}$
↳ does f(x) involve power of x?
- because if $y = a^x$ no matter what x is, y will never be negative

$\{y \in \mathbb{R}, y \geq -4\}$ $\quad$ $\{y \in \mathbb{R}\}$ $\quad$ $\{y \in \mathbb{R}, y \geq 0\}$

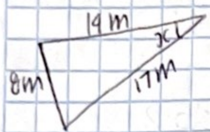
kyla orchard

CHECK CALC - rads or deg

TRIGONOMETRY

~~trigonometry~~

SOHCAHTOA — tan is also $\frac{\sin}{\cos}$

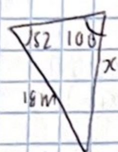


cosine rule

$c^2 = a^2 + b^2 - 2ab\cos C$

$8^2 = 11^2 + 14^2 - 2(17)(14)\cos(x)$

$x = 27.81562176 \approx 28°$

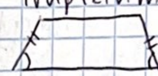


$\frac{x}{\sin(52)} = \frac{18}{\sin(100)}$

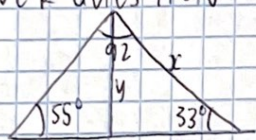
$x = 14.40300761$

$x = 14m$

isosceles trapezium



RIVER QUESTION



$\frac{x}{\sin(55°)} = \frac{120}{\sin(92)}$

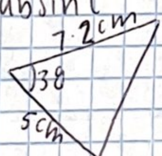
$x = 98.358$

$\sin(33°) = \frac{y}{98.358}$

$y = 53.569 \quad 54m$

AREA

$\frac{1}{2}ab\sin C$

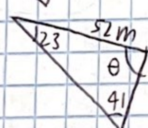


$A = \frac{1}{2}ab\sin C$ — degrees

$= \frac{1}{2}(7.2)(5)\sin(38)$

$= 11.0819$

$= 11.1cm^2$



$180 - 23 - 44 = \theta$

$\theta = 116°$

$A = \frac{1}{2}(52)(41)\sin(116)$

$= 864.6m^2$

RADIANS $\frac{4\pi}{3} \Rightarrow x°$

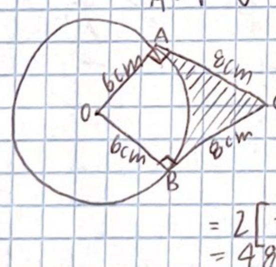
$\frac{\pi}{3} = 60$

60×4

$= 240°$

AREA pt 2.

$A = r^2\theta$ — radians



A of quad - A of sect.

$\tan\theta = \frac{8}{6}$

$\theta = 0.9272$

$\angle AOB = 2 \times 0.9272$

$= 1.8545$

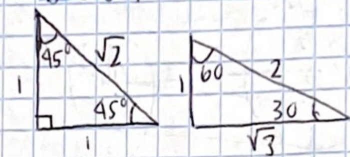
$= 2\left[\frac{1}{2} \times 6 \times 8\right] - \frac{1}{2} \times 6^2 \times 1.8545$

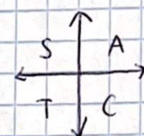
$= 48 - 33.3826$

$= 14.617$

$= 14.6cm^2$

BOOKMARK





$\sin(0°) = 0$

$\sin(90°) = 1$

$\sin(\frac{\pi}{2}) = 1$

$\sin(180°) = 0$

$\sin\pi = 0$

$\cos(0°) = 1$

$\cos(90°) = 0$

$\cos(\frac{\pi}{2}) = 0$

$\cos(180°) = -1$

$\cos(\pi) = -1$

$\theta = rad \times \frac{180}{\pi}$

$rad = \theta \times \frac{\pi}{180}$

$180° = \pi$

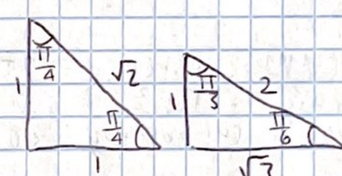
$360° = 2\pi$

$90° = \frac{\pi}{2}$

$30° = \frac{\pi}{6}$

$60° = \frac{\pi}{3}$

$45° = \frac{\pi}{4}$

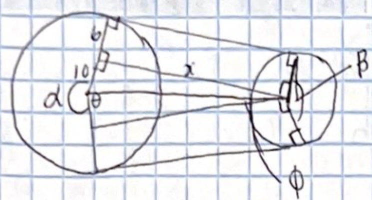


UNIT CIRCLE:

$\cos = x$

$\sin = y$

WHEEL QUESTION



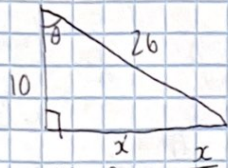
$\tan(\phi) = \frac{10}{24}$

$\phi = 0.39479$

$B = 2\pi - 2\phi = 2\pi - 2(0.3479)$
$= 2.352$

l (major of small) $= 6 \times 2.352$
$= 14.112$

total $= 2x + l(\text{big}) + l(\text{small})$
$= 2 \times 24 + 62.89872 + 14.112$
$= 125\text{cm}$



$\tan\theta = \frac{x}{26}$
$= \frac{24}{26}$

$\theta = 1.176$ radians
$\alpha = 2\pi - 2\theta$
$= 2\pi - 2(1.176)$
$= 3.93117$ radians

l (major of big wheel)
$= r\theta = 16 \times 3.93117$
$= 62.89872$

θ = radians

arc length $= (l = r\theta)$ sector area $= (\frac{1}{2}r^2\theta)$

length of chord $(l = 2r\sin\frac{1}{2}\theta)$ $A = \frac{1}{2}r^2(\theta - \sin\theta)$
area of segment

circumference $= 2\pi r$
area $= \pi r^2$
area of sec or seg $= \frac{\theta}{360} \times 2\pi r$

Kyla Orchard

MATHS NOTES - quadratics

distance formula: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

FEATURES OF QUADRATICS

turning point form: $y=a(x-b)^2+c$

a: nature and dilation of function
$a>0 = \cup$ $a<0 = \cap$
$a>1$ = 'skinny' $0<a<1$ = fat

b: horizontal translation
$-b$ = turning point moves b units right, $+b$ = turning point moves b units left

c: indicates vertical translation
$-c$ = turning point moves c units down, $+c$ = t.p moves c units up

t.p = (b, c) change sign of b, c stays the same

line of sym: $x=b$, change sign of b

y-int: $x=0$ x-int: $y=0$

x-intercept form: $y=a(x-b)(x-c)$ - root form

a: nature and dilation

~~x-ints~~ ~~y~~ = 0, use null factor theorem

line of sym: $x=\frac{b+c}{2}$

t.p: x-coordinate will be answer for line of sym to determine y-coordinate, sub x into original equation + evaluate for y

general form: $y=ax^2+bx+c$

a: nature and dilation

y-int: $x=0$

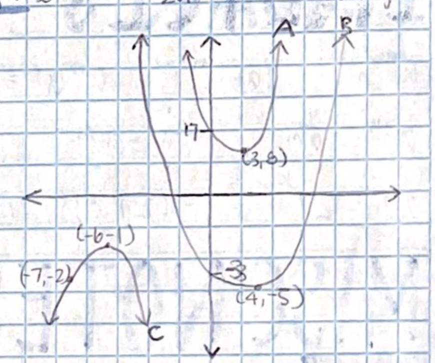
line of sym: $x=\frac{-b}{2a}$ t.p: same as above

x-ints: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ or sub $y=0$

DETERMINING EQUATION FROM GRAPH

~~GRAPH A~~ **t.p form = $y=a(x+p)^2+q$**
- use another point (not t.p)

GRAPH A:	GRAPH B	GRAPH C
$y=a(x+p)^2+q$	$y=a(x+p)^2+q$	$y=a(x+p)^2+q$
$y=a(x-3)^2+8$	$y=a(x-4)^2-5$	$y=a(x+6)^2-1$
sub in (0,17)	sub in (0,3)	sub (-7,-2)
$17=a(0-3)^2+8$	$3=a(0-4)^2-5$	$-2=a(-7+6)^2-1$
$17-8=9a$	$3+5=a(-4)^2$	$-1=a(-7+6)^2$
$\frac{9}{9}=a$	$8=16a$	$-1=1a$
$a=1$	$\frac{8}{16}=a$ $a=0.5$	$\frac{-1}{1}=a$ $a=-1$
$\therefore y=(x-3)^2+8$	$\therefore y=0.5(x-4)^2-5$	$\therefore y=-(x+6)^2-1$

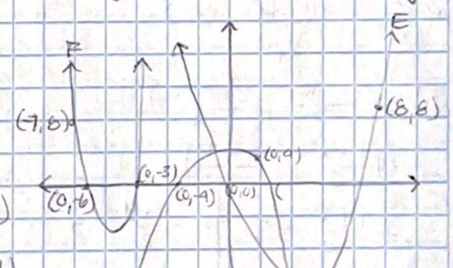


graphs aren't accurate

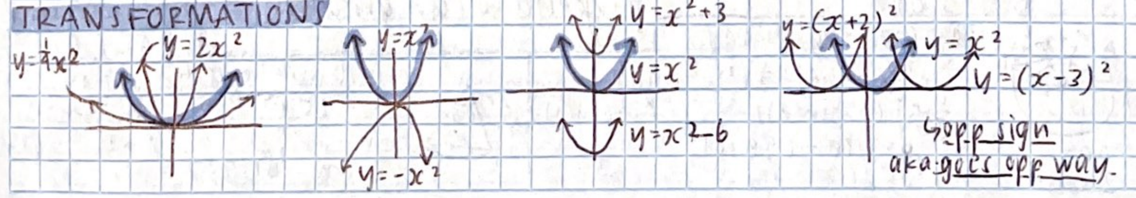
root form: $y=a(x+b)(x+c)$

sub x-coordinates of roots in
- use another known point (not root)

GRAPH E:	GRAPH F:	GRAPH G:
$y=a(x+b)(x+c)$	$y=a(x+6)(x+3)$	$y=a(x+b)(x+c)$
$y=a(x+0)(x-7)$	sub (-7,8)	$y=a(x+4)(x-1)$
$=ax(x-7)$	$8=a(-7+6)(-7+3)$	sub (0,4)
sub (8,8)	$8=4a$	$4=a(0+4)(0-1)$
$8=a(8)(8-7)$	$\frac{8}{4}=a$ $a=2$	$4=-4a$
$8=8a$ $a=\frac{8}{8}=1$	$\therefore y=2(x+6)(x+3)$	$\frac{4}{-4}=a$ $a=-1$
$\therefore y=x(x-7)$		$\therefore y=-(x+4)(x-1)$

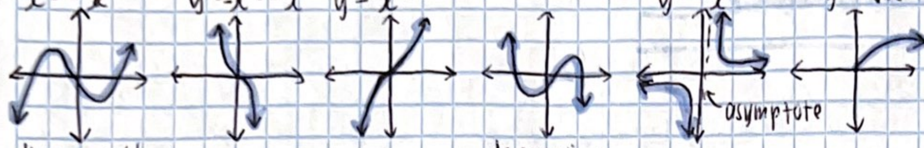


TRANSFORMATIONS



CUBICS ax^3+bx^2+cx+d

$y=x^3-x$ | $y=-x^3-x$ | $y=x^3$ | $y=\frac{1}{x}$ | $y=\sqrt{x}$



relations: 2 y's for same x's
function: 1 y for every x

'normal' positive — negative

if a = positive = ∿ — if a = negative = ∿

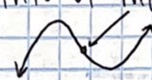
general form: $y=ax^3+bx^2+cx+d$
or $y=ax^3+bx^2+cx$
$y=ax^3+bx$
$y=ax^3$

if $a(x-b)^2(x-c)$ then $(x-b)^2$ = t.p

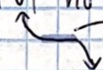
to find x-ints, $y=0$
to find y-int, $x=0$ — must be factorised ↳ use null factor

root form: $y=a(x-b)(x-c)(x-d)$
or $y=a(x-b)^2(x-c)$
$y=ax(x-b)^2$
$y=a(x-b)^3$
$y=ax(x-b)(x-c)$

points of inflection



point of horizontal inflection — is briefly horizontal



example: $-3(x+2)^2(5x+2)$
y-int, $x=0$
$y=-3(0+2)^2(5(0)+2)$
$y=-3(2)^2(2)$
$=-24 \quad \therefore (0,-24)$

x-int/s $y=0$
$0=-3(x+2)^2(5x+2)$
$x+2=0 \qquad 5x+2=0$
$x=-2 \qquad 5x=-2$
$\therefore (-2,0) \qquad (-\frac{2}{5},0)$

INDICES

$a^m b^m=(ab)^m \qquad a^m a^n=a^{m+n} \qquad (a^m)^n=a^{mn}$

$a^{-m}=\frac{1}{a^m} \qquad \frac{a^m}{a^n}=a^{m-n} \qquad a^0=1$

for $a>0$, m an integer and n a positive integer, $a^{m/n}=\sqrt[n]{a^m}=(\sqrt[n]{a})^m$

$27^{\frac{2}{3}}=27^{\frac{2}{1}\times\frac{1}{3}}=(27^{1/3})^2=(3)^2=9$

MISC NOTES

LINEAR

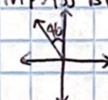
find endpoint given midpoint
e.g. A(1,-4) M(3,2) ? B

$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$
$=\left(\frac{1+x}{2},\frac{-4+y}{2}\right)$
$=\frac{1+x}{2}=3 \quad | \quad =\frac{-4+y}{2}=2$
$=1+x=6 \quad | \quad =-4+y=4$
$x=6-1 \quad | \quad y=8$
$x=5$
$\therefore B=(5,8)$

DIRECT PROPORTION

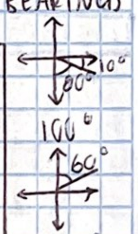
↳ goes through origin
↳ as x increases, y increases

COMPASS BEARINGS

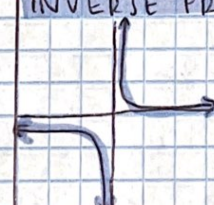


N 46° W

TRUE BEARINGS



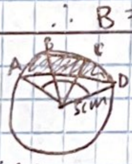
INVERSE PROPORTION



↳ $y=\frac{1}{x}$
↳ as one increases, the other decreases

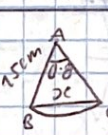
PARALLEL + PERPENDICULAR (LINEAR)

parallel lines have the same gradient
perpendicular lines have gradients that × to −1
perpendicular gradient = flip fraction, change sign



req. area = $seg_{AD}-seg_{BC}$
$=\frac{1}{2}r^2(\theta-\sin\theta)-\frac{1}{2}r^2(\theta-\sin\theta)$
$=\frac{1}{2}\times5^2(2-\sin(2))-\frac{1}{2}\times5^2(1-\sin(1))$
$=13.63370-1.98161$
$=11.6521$
$=11.65cm^2$

$\angle AOD$ = 2 rad



find arc length, $\ell=r\theta$
$=75\times0.8 \quad =60cm$
find x, $x^2=75^2+75^2-2(75)(75)\cos(0.8)$
$x=61.037$

kyla orchard

MATHS NOTES

indices, surds, probability, sequence & series, linear

INDICES

$a^m \times a^n = a^{m+n}$

$(a^m)^n = a^{m \times n}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

$a^m \div a^n = a^{m-n}$

$a^{-m} = \frac{1}{a^m}$

$a^0 = 1$

$(ab)^m = a^m \times b^m$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

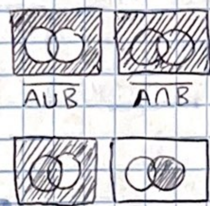
SURDS

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{a} \times \sqrt{a} = a$

$\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

$m\sqrt{a} \pm n\sqrt{a} = (m \pm n)\sqrt{a}$



SET NOTATION

$\bar{A}$ or A' = complement (not)

$A \cup B$ = union (A and B)

$A \cap B$ = intersection (A or B)

$\in$ = element $\notin$ = not element

$\varnothing$ or $\{\}$ = empty set

U = universal set

$\subset$ = subset $A \subset B$ = all in A are in B

$n(A)$ or $|A|$ = no. of elements in A

PROBABILITY LAWS

$P(\bar{A}) = P(A') = 1 - P(A)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) \times P(B|A)$

$P(A \cap B) = P(B) \times P(A|B)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(B|A) = \frac{P(A \cap B)}{P(A)}$

$P(A|B) = \frac{\text{~~all of~~ A in B}}{\text{all of B}}$

ARRANGEMENTS vs COMBINATIONS

ARRANGEMENTS: AB & BA = different; number of terms; factorial (i.e 3!)

$\frac{100!}{99!} = 100$ (cancels)

COMBINATIONS: AB & BA = same

$^nC_r = \frac{n!}{r!(n-r)!}$

n = how many you have

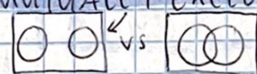
r = how many you want

e.g $^6C_4 = \frac{6!}{4!2!}$

or on calc: nCr (6,4)

$^nC_r = \binom{n}{r}$

MUTUALLY EXCLUSIVE

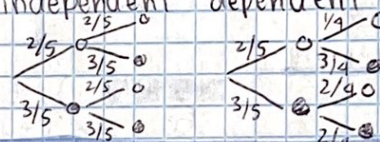


$A \cap B = 0$

INDEPENDENCE

independent: occurrence of one doesn't effect the other

independent dependent



MUTUALLY EXCLUSIVE RULES

$P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B)$

INDEPENDENT EVENTS RULES

$P(A \cap B) = P(A) \times P(B)$

$P(A|B) = P(A)$

$P(B|A) = P(B)$

SEQUENCE NOTATION

T_n = n^{th} term

a = initial term

d = common difference

r = common ratio

S_n = sum of first n terms

S_∞ = sum to infinity

ARITHMETIC vs GEOMETRIC

ARITHMETIC

$d = T_{n+1} - T_n$ or $d = T_2 - T_1$

explicit: $T_n = a + (n-1) \times d$

recursive: $T_{n+1} = T_n + d,\ T_1 = a$

sum: $S_n = \frac{n}{2}(2a + (n-1) \times d)$

$S\infty$: $S_\infty = \infty$ or $-\infty$

GEOMETRIC

$r = T_{n+1} \div T_n$ or $r = T_2 \div T_1$

explicit: $T_n = a \times r^{n-1}$

recursive: $T_{n+1} = T_n \times r,\ T_1 = a$

sum: $S_n = \frac{a(1-r^n)}{1-r}$ for $-1 < r < 1$

or $\frac{a(r^n - 1)}{r-1}$ for $r > 1, r < -1$

$S\infty$: $S_\infty = \frac{a}{1-r}$

GROWTH & DECAY

growth: $T_n = a \times (1+r)^t$

$T_{n+1} = (1+r) \times T_n,\ T_1 = a$

decay: $T_n = a \times (1-r)^t$

$T_{n+1} = (1-r) \times T_n,\ T_1 = a$

e.g recursive formula to model 8 rabbits growing 40% per year ☆

$a = 8,\ r = 40\% = 0.4$

$T_{n+1} = (1 + 0.4) \times T_n$

$\therefore T_{n+1} = 1.04\,T_n,\ T_1 = 8$

PARALLEL LINES

$m_1 \times m_2 = -1$

$m_1 = \frac{1}{m_2}$

~~PARA~~ MIDPOINT

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

ENDPOINT

$(2x_1 - x_2, 2y_1 - y_2)$

midpoint co-ords

endpoint

MISC.

distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$S_\infty = \frac{a}{1-r}$

$\frac{r}{1} = \frac{T_2}{T_1}$ $a = \frac{T_2}{r}$

$S_\infty = \frac{T_2}{r} \div \frac{1-r}{1}$

$T_{10} = S_{10} - S_9$

quadratics, bearings, domain & range, circles

general form

$y = ax^2 + bx + c$

a = concavity/dilation

$a > 0$ = ∪ shape

$a < 0$ = ∩ shape

c = ~~y-int~~ vertical translation

line of sym: $x = \frac{-b}{2a}$

t.p = sub into original

x-ints = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

or $y = 0$

t.p form

$y = a(x-b)^2 + c$

a = nature/dilation

b = horizontal translation

t.p = (b, c), change sign of b, c = same

line of sym = b (change sign)

y-int, $x = 0$, x-ints $y = 0$

x-int form/root form

$y = a(x-b)(x-c)$

a = nature/dilation

line of sym = $\frac{b+c}{2}$

t.p = use ↑ solve for y

x-ints = $y = 0$, NFT

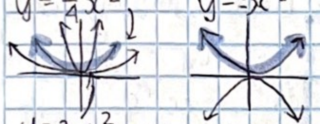
DETERMINE EQUATION FROM GRAPH

t.p form – use another point (not t.p)

root form – sub x-coords in, use another known point (not root)

TRANSFORMATIONS

$y = \frac{1}{4}x^2$ $y = x^2$



$y = 2x^2$

DISCRIMINANT

$\Delta = b^2 - 4ac$

$\Delta > 0$ = 2 roots

$\Delta = 0$ = 1 root

$\Delta < 0$ = no roots

FIND ENDPOINT GIVEN MID

LINEAR

$A(1, -4)$ $M = (3, 2)$ $B = ?$

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \frac{1+x}{2} = 3$ | $\frac{-4+y}{2} = 2$

$x = 5$ | $y = 8$

$B = (5, 8)$

EXPONENTIAL

$y = a^x$

reflect = $y = a^{-x}$

QUADRATIC CONVERSIONS

x-int form $\xrightarrow{\text{expand}}$ general form $\xrightarrow{\text{complete the square}}$ t.p form

t.p form $\xrightarrow{\text{expand}}$ general form $\xrightarrow{\text{factorise}}$ x-int form

☆ **completing the square e.g**

$y = 2x^2 - 20x - 42$

$2(x^2 - 10x - 21)$

$y = 2\left[\left(x - \frac{10}{2}\right)^2 - \left(-\frac{10}{2}\right)^2 - 21\right]$

$y = 2[(x-5)^2 - 25 - 21]$

$y = 2(x-5)^2 - 92$

DOMAIN $\{x \in \mathbb{R}\}$

$\sqrt{x}$ $x \neq$ negative

$\frac{1}{x}$ $x \neq 0$

RANGE $\{y \in \mathbb{R}\}$

$\sqrt{x}$ or $\frac{1}{x}$

$y \geq$

CIRCLES

$(x+4)^2 + (y-1)^2 = 3^2$

flip signs = mid   radius

e.g $x^2 + y^2 + 6y = 10x$

$x^2 - 10x + 25 + y^2 + 6y + 9 = 0 + 25 + 9$

$(x-5)^2 + (y+3)^2 = 34$ ☆

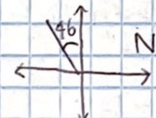
$= (\sqrt{34})^2$

centre @ $(5, -3)$

radius = $\sqrt{34}$

$\tan = \frac{\sin}{\cos}$

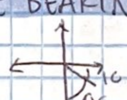
COMPASS BEARINGS



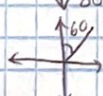
N46°W

								1								
							1		1							
						1		2		1						
					1		3		3		1					
				1		4		6		4		1				
			1		5		10		10		5		1			
		1		6		15		20		15		6		1		
	1		7		21		35		35		21		7		1	
1		8		28		56		70		56		28		8		1

TRUE BEARINGS



100°



060°

CHECK CALC Rad or deg

trends:

- on rows beginning w/ primes, all no. are divisible by first no.
- add no. on a row = power of 2
  row 1 = 1 = 2^0, row 3 = 4 = 2^2
- put all no. together = power of 11
  row 2 = 11^1, row 3 = 1331 = 11^3, row 4 = 11^3

combinations

^n_r n row #, r column #   e.g $^4_2 = 6$

expand $(x+y)^5$

check row 1 after

$1x^5y^0 + 5x^4y^1 + 10x^3y^2 +$

☆ $10x^2y^3 + 5xy^4 + y^5$

x's go down

y's go up

$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3$

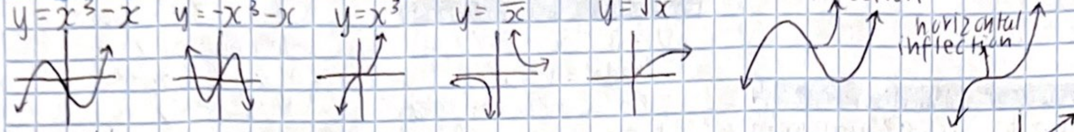
$+ 5xy^4 + y^5$

kyla orchard

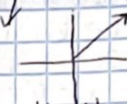
MATHS NOTES

cubics, trig, area of triangles, circular measure

$y=x^3-x$ $y=-x^3-x$ $y=x^3$ $y=\frac{1}{x}$ $y=\sqrt{x}$



'normal' (positive)



direct proportion (thru 0,0)

general form

$y=ax^3+bx^2+cx+d$

if $a(x-b)^2(x-c)$ then $(x-b)^2 = \pm p$

x-ints, $y=0$ y-ints, $x=0$

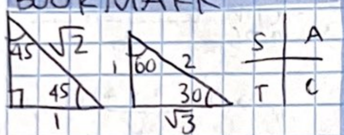
root form

$y=a(x-b)(x-c)(x-d)$

y-int, $x=0$

x-ints, $y=0$

BOOKMARK



$\sin(0)=0$

$\sin(90)=1$

$\sin(\frac{\pi}{2})=1$

$\cos=x$

$\sin=y$

unit circle

$\sin(180)=0$

$\sin\pi=0$

$\cos(0)=1$

$\cos(90)=0$

$\cos(\frac{\pi}{2})=0$

$\cos(180)=-1$

$\cos(\pi)=-1$

$\theta = rad \times \frac{180}{\pi}$

$rad = \theta \times \frac{\pi}{180}$

$180° = \pi$

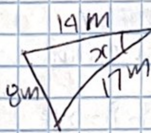
$360° = 2\pi$

$90 = \frac{\pi}{2}$

$30 = \pi/6$

$60 = \pi/3$

$45 = \pi/4$

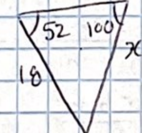


cos rule

$c^2=a^2+b^2+2ab\cos C$

$8^2=11^2+14^2-2(17)(14)\cos(x)$

$x=27.81 \approx 28°$



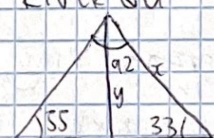
sine rule

$\frac{x}{\sin(52)} = \frac{18}{\sin(100)}$

$x = ~~98.358~~$ 14

$\approx x = 14m$

RIVER QU.



$\frac{x}{\sin(55)} = \frac{120}{\sin(92)}$

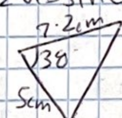
$x = 98.358$

$\sin(33) = \frac{y}{98...}$

$y = 53.569 = 54m$

AREA

$\frac{1}{2}ab\sin C = area$



$A=\frac{1}{2}ab\sin C$ — degrees

$=\frac{1}{2}(7.2)(5)\sin(38)$

$=11.0819 = 11.1cm^2$

RADIANS

$\frac{4\pi}{3} \rightarrow x°$

$\frac{\pi}{3} = 60$

$60\times4 = 240°$

CIRCULAR MEASURE

length of arc

rad $= r\theta$

deg $= r(\frac{\pi\theta}{180})$

area of sector

rad $= \frac{1}{2}r^2\theta$

deg $= \frac{1}{2}r^2(\frac{\pi\theta}{180})$

length of chord

$(2r\sin\frac{1}{2}\theta)$

area of segment (sector - triangle)

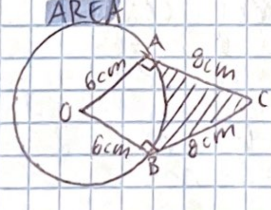
rad $= \frac{1}{2}r^2(\theta - \sin\theta)$

deg $= (\frac{1}{2}r^2[(\frac{\pi\theta}{180}) - \sin(\frac{\pi\theta}{180})]$

area of seg

$A=\frac{1}{2}r^2(\theta-\sin\theta)$

CIRCLE-RELATED AREA



$A = r^2\theta \rightarrow$ radians

A of quad A of sect

$\tan\theta = \frac{8}{6}$

$\theta = 0.9272$

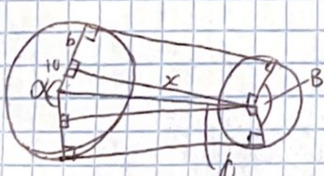
$<AOB = 2x = 1.8545$

$2[\frac{1}{2}\times6\times8] - \frac{1}{2}\times6^2\times1.8545$

$=48-33.3826$

$=14.617$

$=14.6cm^2$



$\tan(\phi) = \frac{10}{24}$ $\phi = 0.39479$

$B = 2\pi - 2\phi$

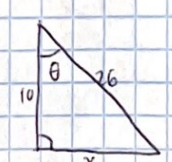
$= 2\pi - 2(0.3479) = 2.352$

ℓ (major of small) $= 6\times2.352$

$=14.112$

total $= 2x + \ell(big) + \ell(small)$

$= 2\times24 + 62.89872 + 14.112 = 125cm$



calculus, pain, sequence again

$y = ax^n$
$dy/dx = anx^{n-1}$

co-ords of given gradient

when on $y=x^2$ will $m=4$?
$y = x^2$
$dy/dx = 2x$ $dy/dx = 4$
solve $x = 2$
sub $x=2$ into original
$y = 4$ $\therefore (2,4)$

ON CLASSPAD

-tangent (sketch)
⟵ ⟶ to move
in main
math2 $\frac{d}{d\square}\square$
use | symbol to evaluate dy/dx for x

OPTIMISATION

① reduce variables to 2 by substitution + simplification
② find dy/dx
③ $dy/dx = 0$
④ solve for t.ps
⑤ ~~xxx~~ nature test

RECTILINEAR MOTION

displacement
differentiate ⟶ velocity

ANTI-DIFFERENTIATING

$$f(x) = \int f'(x)\,dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

don't forget

equation of tangent

$y = 0.5x^3$ @ $(2,4)$
$dy/dx = 1.5x^2$
sub $x=2$ into $\frac{dy}{dx}$
$y = 6x + c$ ↑find m
find c = sub $(2,4)$ into $y = 6x + c$ + solve
$\therefore y = 6x - 8$

$$\frac{\delta y}{\delta x} = \frac{f(3)-f(2)}{3-2}$$

eg $f(x) = 3x^2 - 4x + 1$
when $x = 2$
$f'(x) = 6x - 4$
sub $x = 2$
$m = 8$ $\therefore y = 8x + c$
sub x into original
$y = 5$
find cutting point
$y = 8x - 11$

LIMITING THEOREM

prove gradient function of $x^2 = 2x$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim \frac{(x+h)^2 - x^2}{h}$$

$$\frac{x^2 + 2hx + h^2 - x^2}{h}$$

$$\frac{2hx + h^2}{h}$$

$$\frac{\cancel{h}(2x+h)}{\cancel{h}}$$

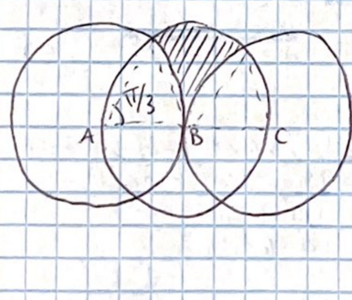
$= 2x + h$
sub $h = 0$
$\therefore 2x$

FIND T_1

~~given~~ $\overline{18}$ — $\overline{-486}$
① find r
$-486 = 18 \times r^3$ $r = -3$
$\therefore T_1 = 18 \div (-3)^2$ ← back 2
$= 2$

given jumps × d + given
= d given −
then given d × given = T_1

$T_{19} = 61$ $T_{41} = 127$
$61 + 22d = 127$
$d = 3$
$T_1 = 61 - 18(3)$
$= 61 - 54$
$= 7$



perimeter
= P = 3 × arc length
$3 \times 6 \times \frac{\pi}{3}$
$= 18.85$cm
=

A sector (centre B) segment (centre A) − segment (C)

$= \frac{1}{2} r^2\theta$

$= 18\sqrt{3} - 6\pi\ \text{cm}^2$
$= 12.33\text{cm}^2$

~~calculus - intro~~ CALCULUS - INTRO

THE POWER RULE

if $y = ax^n$

then $\frac{dy}{dx} = anx^{n-1}$

e.g. $y = x^2$

$\frac{dy}{dx} = 2x$

CALCULATING THE GRADIENT AT A POINT - SUBSTITUTING

- a.k.a the instantaneous rate of change at a particular point

e.g. what is the gradient of the curve $y = x^2 - 3x$ @ the point $(2, -2)$?

$y = x^2 - 3x$

$\frac{dy}{dx} = 2x - 3$

sub $x = 2$ into dy/dx

$= 2(2) - 3$

$\frac{dy}{dx} = 1$

STEPS:

1. differentiate original function
2. sub x value of point into dy/dx function
3. dy/dx is same as m ∴ answer

DETERMINING THE CO-ORDINATES OF A GIVEN GRADIENT - SOLVING

e.g. at what points on the curve $y = x^2$, will the gradient be equal to 4?

$y = x^2$

$\frac{dy}{dx} = 2x$

sub $\frac{dy}{dx} = 4$ (+ solve)

$4 = 2x$

$x = 2$

to determine y-value of the point sub $x = 2$ into original function + evaluate for y

$y = (2)^2$

$= 4$ ∴ $(2, 4)$

STEPS:

1. find dy/dx
2. sub gradient into dy/dx, find x
3. find y by plugging x into original equation

DETERMINE THE EQUATION OF THE TANGENT - SUBSTITUTION + SOLVING

- tangent is the straight line that intersects the curve

e.g. determine the equation of the tangent to the curve $y = 0.5x^3$ at the point $(2, 4)$

$y = 0.5x^3$

$\frac{dy}{dx} = 1.5x^2$

gradient @ $(2,4)$ → sub x into dy/dx function

$y = 1.5(2)^2 = 6$

∴ $y = 6x + c$

find c by subbing $(2, 4)$ into $y = 6x + c$ and solving

$4 = 6(2) + c$

$4 - 12 = c$

$c = -8$ ∴ $y = 6x - 8$

STEPS:

1. find dy/dx
2. sub x into dy/dx, find m
3. find c by subbing point into $y = mx + c$
4. add c to $y = mx + c$ = answer

e.g. determine the equation of the line that is tangential to the curve $f(x) = 3x^2 - 4x + 1$ when $x = 2$ (not y co-ord)

$f(x) = 3x^2 - 4x + 1$

$f'(x) = 6x - 4$

m when $x = 2$, sub $x = 2$

$f'(2) = 6(2) - 4$

$= 8$ ∴ $y = 8x + c$

to find y, sub x into original

$3(2)^2 - 4(2) + 1$

$= 12 - 8 + 1$ $= 5$

find c using point $(2, 5)$

$5 = 8(2) + c$

$5 - 16 = c$

$c = -11$ ∴ $y = 8x - 11$

ON CLASSPAD

Graphs + Tables

- tangent (under sketch)

use ← and → arrows to move line

Main

- math2, dy/dx button $\frac{d}{d\square}\square$ to determine gradient function
- use | symbol to evaluate for a given x value
- use (solve) to solve and find x value/s

UNDERSTANDING + USING THE DIFFERENCE QUOTIENT

average rate of change

$= \frac{\delta y}{\delta x}$ ← change in
lowercase Δ

e.g. determine the average rate of change from f(2) to f(3)

$f(2) = 4$ → $(2,4)$ $f(3) = 9$ → $(3,9)$

$= \frac{\delta y}{\delta x}$

$= \frac{f(3) - f(2)}{3 - 2} = \frac{9-4}{3-2}$

$= \frac{5}{1} = 5$

determine the gradient (i.e the instantaneous rate of change) at f(2)

$\frac{dy}{dx} = 2x$

gradient @ $x = 2$

$dy/dx = 2(2)$

$= 4$

use: $\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

to prove that the gradient function of $y = x^2$ is $2x$

$\frac{dy}{dx} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$ ← sub in f() of equation you're using

$= \lim_{h \to 0} \frac{\cancel{x^2} + 2hx + h^2 - \cancel{x^2}}{h}$

$= \lim_{h \to 0} \frac{2hx + h^2}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(2x + h)}{\cancel{h}}$

$= \lim_{h \to 0} 2x + h$

sub 0 into h since $h \to 0$

$2x + 0$

$\frac{dy}{dx} = 2x$

∴ gradient function $= 2x$

calculus - applications

RATES OF CHANGE

$y = 2x + 1$
would be $\frac{dy}{dx}$
but $v = 2t + 1$
would be $\frac{dv}{dt}$

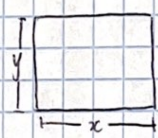
LOCATE STATIONARY POINTS

$\frac{dy}{dx} = 0$, then solve for x.
sub x into original

OPTIMISATION

example
what should the dimensions of a rectangular shape of perimeter 20cm if its area is to be a max.?

solution:



Area = A cm^2
Area(A) = xy
differentiate A because the right side has 2 variables (x and y)
we also know $2x + 2y = 20$cm
$\therefore y = 10 - x$ (swap to)
sub $A = xy$ ~~____~~ $y = 10 - x$
$A = 10x - x^2$
differentiate:
$\frac{dA}{dx} = 10 - 2x$
if $\frac{dA}{dx} = 0$ then:
$0 = 10 - 2x$
$\therefore x = 5$
by inspection of the graph function, the negative coefficient of x^2 shows it is a ∩ graph
$\therefore x = 5$ gives a max value of A
+ when ~~y~~ = 4, $y = 5$ $\therefore$ the lengths should be 5cm × 5cm

help to solve optimisation qu's

① draw a graph
② identify what needs to be maximised/minimised
③ if the equation you find has 2 variables, find another to sub in
④ when you have C in terms of 1 variable, say x, then find $\frac{dc}{dx} = 0$
⑤ figure out whether it's a max or min by inspection of the equation
⑥ check domain/range!!

*cannot differentiate when 2 variables!!

ANTIDIFFERENTIATION

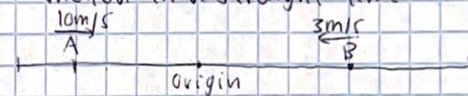
i.e integrals: ∫

if $\frac{dy}{dx} = anx^{n-1}$
then $y = \frac{anx^n}{n}$
$= ax^n$

$\therefore \frac{dy}{dx} = ax^n$ then $y = \frac{ax^{n+1}}{n+1} + C$ ← don't forget!

RECTILINEAR MOTION

← motion in a straight line



A - distance = 4m, displacement = -4m
speed = 10m/s, velocity = 10m/s

B - distance = 5m, displacement = 5m
speed = 3m/s, velocity = -3m/s

if displacement = x metres
then $\frac{dx}{dt}$ = velocity

if given a displacement ($\frac{dx}{dt}$) function, differentiate to get velocity ← $v = \frac{dx}{dt}$
x in terms of t

velocity ⟶ displacement = antidifferentiate

kyla orchard

~~sequences + series~~ SEQUENCES + SERIES

SEQUENCES

AP - arithmetic progression

$T_{n+1} = T_n + d$, $T_1 = a$

$T_n = a + d(n-1)$

to determine constant difference: $T_{n+1} - T_n$

GP - geometric progression

$T_{n+1} = rT_n$, $T_1 = a$

$T_n = ar^{n-1}$

to determine constant difference: $r = \frac{T_{n+1}}{T_n}$

SERIES

arithmetic sequences

for initial term, a, and difference, d:

$T_n = a + (n-1)d \quad n \geq 1$

$T_{n+1} = T_n + d$, where $T_1 = a$

$S_n = \frac{n}{2}(2a + (n-1)d)$

geometric sequence!

for initial term, a, and common ratio, r:

$T_{n+1} = rT_n$, where $T_1 = a$

$T_n = ar^{n-1}$, $n \geq 1$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_\infty = \frac{a}{1-r}$, $|r| < 1$

FINDING TERM 1, GIVEN OTHER TERMS

e.g. $T_3 = 18$, $T_6 = -486$ geometric

__ __ 18 __ __ -486

$-486 = 18 \times r^3$ ← 3 jumps right

$r = -3$

$\therefore T_1 = 18 \div (-3)^2$

$= 2$

e.g. $T_{19} = 61$, $T_{41} = 127$ arithmetic

61 (T_{19}) ... 127 (T_{41})

$61 + 22d = 127$ ← distance

$d = 3$

$\therefore T_1 = 61 - 18(3)$

$= 61 - 54$

$= 7$

SOMETHING RANDOM I WROTE

$S_\infty = \frac{a}{1-r}$

$\frac{r}{1} = \frac{T_2}{T_1}$ $\quad a = \frac{T_2}{r}$

$S_\infty = \frac{T_2}{r} \div \frac{1-r}{1}$

$T_{10} = S_{10} - S_9$

EXAMPLE QUESTIONS

Georgia rents her house out at \$300 per week. Each year, she increases the weekly rent by \$20. determine how long it would take for the weekly rent to increase by 80%.

$300 \times 1.8 = 300 + n(20)$

$n = 12$

$\therefore$ will take 12 years

Rebecca invests \$250,000 in an account that pays interest at a rate of 5.2% per annum, compounded annually. How many years will it take for the balance to increase by at least \$500,000

$500{,}000 = 250{,}000(1+0.052)^t - 250{,}000$

$t = 21.6718$

≈ 22 $\therefore$ 22 years